**Module 3**

**Analysis of function using the derivative, plotting graph of function.**

**Lecture 11.**

**L’Hospital’s Rule. Analyzing the growth of functions using . The Indeterminate Forms. Taylor’s formula. Remainder.**

**L’Hospitals Rule**

***Evaluating the indeterminate forms*  and **

Let the single-valued functions  and  be differentiable for  the derivative of one of them does not vanish. If  and  are both infinitesimals or both infinities as  that is, if the quotient  at , is one of the indeterminate forms  or  then

 

provided that the limit of the ratio of derivatives exists.

The rule is also applicable when 

If the quotient  again has as indeterminate form, at the point , of one of the two above-mentioned types and  and  satisfy all the requirements that have been stated for  and , we can then pass to the ratio of second derivatives, etc.

However, it should be borne in mind that the limit of the ratio  may exist, whereas the ratios of the derivatives do not tend to any limit.

 **Example.** Compute:

 ;

 **Solution:** Applyingthe L’Hospital rule we have

 ;

**Other indeterminate forms.** To evaluate an indeterminate form like

 , transform the appropriate product , where  into the quotient

 

 In the case of indeterminate form $(\infty -\infty )$, reducing to a common denominator we get the indeterminate form 

 The indeterminate forms $1^{\infty }, 0^{0}, \infty ^{0}$ are evaluated by first taking logarithms and then finding the limit of the logarithm of the power $f\_{1}(x)^{f\_{2}(x)}$ (which requires evaluating a form like ).

 In certain cases it is useful to combine the L’Hospitals rule with the finding of limits by elementary techniques.

**Taylor’s Formula**

If a function  is continuous and has continuous derivatives up to

the  order inclusive on the interval $a\leq x\leq b, (or b\leq x\leq a)$ , and there is a finite derivative $f^{n}(x)$ at each interior point of the interval, then *Taylor’s formula*

 (1)

where

 (2)

holds true on the interval. Formula (2) is *Lagrange formula for remainder*.

In particular, when $x\_{0}=0$ we have *Maclaurin’s formula*.



where 